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Democracy in Product Design: Consumer Participation and Differentiation Strategies*

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Democracy in Product Design: Consumer Participation and Differentiation Strategies

Abstract

An increasing number of firms use social media to allow their customers to vote on new product designs. This paper studies the implications of employing such a democratic product design (DPD). A linear city model is used with random locations to capture uncertainty about consumer preferences and to study strategic forces in monopoly and duopoly settings. The results indicate that a monopolist will use market research to resolve the demand uncertainty, unless DPD provides a cost advantage. In a duopoly, an asymmetric equilibrium emerges with exactly one firm using DPD. Commitment to following consumer votes proves to be a strategic advantage, therefore at least one firm promises not to deviate from the product design consumers voted for. A subtle way to influence the outcome of the vote for firms is to generate product candidates instead of soliciting ideas from consumers. Employing such a tactic allows firms to differentiate and they will be more likely to use DPD. Finally, the paper studies the level of consumer engagement in DPD and shows that a monopolist always benefits from a higher positive engagement and is hurt by negative engagement, although to a lesser extent. The results are reversed for a duopolist as negative sentiments can serve as an additional differentiator.

Keywords: *co-creation; consumer voting; new products; strategic differentiation*

1 Introduction

There is an increasing number of firms inviting consumers to be closely involved in new product decisions. These firms allow consumers to vote about products in a democratic fashion and then implement and produce the winning candidate. For example, Mountain Dew released its brand new “White Out” product designed through the “Dewmocracy 2” campaign which involved consumers in decisions on the flavor, color, and the name of the product. In the final round, consumers voted for one of three versions of the soft drink to be produced and sold in stores. The smooth citrus flavored White Out received 44% of the votes, surpassing the strawberry-pineapple flavored Typhoon (40%) and the lime flavored Distortion (16%).¹ In a related product category, Coca-Cola’s Vitamin Water preceded Mountain Dew (although not in the carbonated soft drinks category) with its “flavorcreator” Facebook application that let consumers design and vote for the next flavor of the drink.² Other examples include M&M’s soliciting over ten million votes for the new color of candy to be included in their bags (Fuchs et al. 2010) or the DoUsAFlavor site by Lay’s Chips letting consumers vote on different new flavors. Similarly, the Cheesecake Factory announced it’s new Stefanie’s Ultimate Red Velvet Cheesecake at the conclusion of the “What’s Your Flavor?” vote (Kim et al. 2014), while in Europe, Danone led the way by accepting more than six million consumer votes online for the next flavor of its Danette brand in 2006.

Customer involvement in new product development is not a radically new phenomenon, especially in the B2B context, but since the advent of social media there has been a recent surge in consumer-controlled product introductions. Even without a formal voting process or “election” in place, many brands encourage their customers to participate in shaping the brand’s image and to contribute with new products ideas. Often, a brand’s Facebook fan page is an important venue where consumers can make comments or participate in polls³ that relate to the future of the brand. Although there are various potential advantages of such deep consumer involvement (Prahalad and Ramaswamy 2000, Fuchs et al. 2010), it is interesting to see that a company gives almost exclusive control to its customers even in the highly competitive soft drink industry, where brand positioning and product differentiation have always been important (Yoffie and Kim 2010).

A crucial component of social media that has an important effect on these situations is *transparency*.

¹ “What Mountain Dew Learned from DEWmocracy”, Adweek (2010) June, www.adweek.com.

² “Crowdsourcing 101: Why Vitaminwater’s Facebook App Can’t Lose”, www.fastcompany.com.

³ “Facebook fine-tunes its Questions product”, (2011) March news.cnet.com

In most cases consumers can follow the vote or poll results on the web or on social media which makes it hard for firms not to implement the vote outcome. By deviating from what consumers voted for, firms risk upsetting their current or future customers. While in earlier cases of consumer participation only a small number of consumers may have been involved and the process could have been conducted in relative isolation, social media potentially enables every single consumer to follow the process.

The goal of this paper is to examine the effects of consumer participation in product design on firm strategies, in particular with respect to competition and differentiation. Although the number of marketers employing such tactics has been on the rise, there are many that shy away from involving consumers so deeply. Therefore, it is important to understand the key tradeoffs driving a firm's choice to involve consumers in product related decisions. We focus our attention on what we call democratic product design (DPD). In a DPD process, participating consumers collectively decide on a new product's features, often through a vote. The firm organizing the vote then implements the decision or otherwise it faces potentially disappointed consumers with a lower willingness to pay for its product.

Having consumers vote about a new product clearly benefits the firm by revealing consumer preferences. The key question is whether giving so much control to consumers interferes with the firm's fundamental strategic goals, such as differentiating its products. While a monopolist may not worry about differentiation, competing firms could be significantly affected by such high level of consumer control. An important question is whether following the majority tastes potentially hurts those firms that would like to differentiate or does it provide strategic advantages? At the same time, how is consumer welfare affected?

Consumer involvement in the product design often involves an implicit promise that the firm will honor the votes. However, given the strategic decisions at stake, is it credible that firms will follow consumer votes? Or do firms have an incentive to deviate from the design that consumers voted for? Finally, involving consumers may have additional marketing benefits through consumer engagement with the product or the brand. We explore how consumer engagement enticed by the participation affects firm strategies and whether increased consumer engagement is always beneficial for firms.

We develop an analytical model to address these questions, building on the standard horizontal differentiation framework, but introducing consumer voting and uncertainty in consumers' locations. As in a typical linear city, consumers are located in the $[0, 1]$ interval, but we assume that firms do not know whether they lean more towards the left or the right side of this interval. We then allow

firms to make a choice between choosing their location based on their priors, using traditional market research, or engaging in a democratic product design process which allows consumers to vote on the final product.

We find that unless DPD carries a cost advantage or provides direct benefits to participating consumers, a monopolist will always want to use traditional market research to *learn* about consumer preferences without having to *commit* to following consumer votes. Interestingly, the results are starkly different in a duopoly. In equilibrium, one firm will use a democratic design process, whereas the other firm will use traditional market research. The reason for the asymmetric equilibrium is that although both market research and involving consumers presents an opportunity to *learn* consumer preferences, *commitment* plays a crucial role in democratic product design. When only one firm commits to follow average consumer preferences, its competitor is forced out of the center of the market, leaving the first firm in a dominant position while allowing for differentiation and relatively soft price competition. On the other hand, when both firms commit to follow consumer votes that are likely to be in the center of the market, they will find themselves in fierce price competition. In other words, using a democratic design process acts as a deterrent to enter the middle of the market yielding superior profits for the firm that engages in it, but preventing its competitor from also doing so. Allowing for mixed strategies, we find a corresponding unique symmetric equilibrium where each firm uses consumer voting with a certain probability close to $1/2$.

Although the asymmetric equilibrium is beneficial for the firm using DPD by ensuring high profits, it is important to compare the profitability of the industry and consumer surplus between different scenarios. The total industry profits (and thus the profit a firm can expect if it is randomly assigned a role in the asymmetric equilibrium) are strictly lower than if both firms used traditional market research or neither of them used market research. The expected profits are further reduced in the symmetric mixed strategy equilibrium as there is a chance of very intense price competition if both firms use DPD. Therefore, firm profits are clearly hurt by the possibility of using DPD due to the strategic effects of commitment. In contrast, consumer surplus increases as a result of DPD as consumers are better off both in the asymmetric equilibrium and the symmetric mixed equilibrium. This is not only a result of products in the market that fit the average consumer's taste better, but also a result of increased price competition.

To further examine the role of commitment, we analyze firms' incentives to follow consumer votes and to manipulate the results. We extend the model by allowing firms to deviate from the vote

outcome at the expense of being penalized by consumers. When the penalty is high we replicate our main results, but at lower levels of consumer sensitivity, a different equilibrium emerges in which both firms use DPD and deviate from the results to reach a moderate level of differentiation. This results in lower profits for both firms than if firms used only market research. We also investigate whether firms would want to impose a penalty upfront by making promises to consumers about following the vote results. We find that if firms have a way to self-impose a high enough penalty at least one of them will do so. This suggests that at least one firm has an incentive to implement a transparent process, thereby committing to not manipulating the results of the vote. Interestingly, when the maximum level of transparency limits the level of commitment firms can make, they will both use DPD anticipating not to deviate from the vote results, but this leads to tough price competition and reduced prices.

Finally, we examine the role of consumer engagement. Often, an increased benefit of involving consumers in product design is that they spend time on the firm’s website or social media page, possibly improving their attitude and willingness to pay for a product. We find that this is always beneficial for a monopolist, but may be detrimental in a duopoly as it makes consumer voting too attractive to firms, hindering their ability to effectively differentiate. Furthermore, we analyze the potential backlash from consumers who are upset if they voted for a product design that did not win. Surprisingly, such a negative reaction typically does not hurt duopolists as it acts as an extra differentiator.

2 Relevant Literature

The idea of involving consumers in production emerged in the literature in the B2B context (von Hippel 1976) and was further developed with a focus on the role of lead users in developing new product ideas (von Hippel 1986). In a more general sense, the concept of co-creation, introduced by Prahalad and Ramaswamy (2000), acknowledges the economic value of involving customers from conception to delivery of the product. The subsequent literature consists of papers developing the framework of co-creation (Prahalad and Ramaswamy 2004, Payne et al. 2008), and open innovation (Chesbrough 2003), considering its psychological implications (Bendapudi and Leone 2003, Fuchs et al. 2010, Kim et al. 2014), examining the role of the Internet (Mohanbir Sawhney 2005), the effect on consumer loyalty (Auh et al. 2007), and the process of eliciting ideas from consumers (Morgan and Wang 2010). Our paper has a substantially different focus from the existing literature in that we concentrate on the competitive effects of consumer involvement in product design. In particular, we

study a B2C context with wide consumer participation and examine consumers' role only in product design and not in production or delivery with a special focus on competition.

We use a standard horizontal differentiation model with location choice (Hotelling 1929, D'Aspremont et al. 1979). Important details of the location choice decision in a linear city have been uncovered by de Palma et al. (1985), Economides (1986), Rhee et al. (1992), Rath and Zhao (2001), Gal-Or and Dukes (2003) who have all analyzed conditions that affect how much firms want to differentiate. In the basic model of D'Aspremont et al. (1979), firms have an incentive to move towards the extremes (the principle of maximum differentiation), but under certain conditions – when consumer heterogeneity is large enough, when the curvature of travel cost function is not very high, when important product attributes are unobserved by the firms, or when demand is elastic – they will locate closer to one another. Our model sheds light on how consumer participation affects the level of differentiation as the decision to involve consumers can be thought of as a move towards less differentiation.

In the marketing literature a number of papers, such as, Iyer and Soberman (2000) and Lauga and Ofek (2009) study the role of market research in resolving demand uncertainty. In our model, we compare the role of traditional market research and DPD under a specific type of demand uncertainty, in the linear city setup. In their two papers Meagher and Zauner (2004, 2005) treat uncertainty in a similar way, although we use a different distribution. Król (2011) generalizes the measure of uncertainty, but also finds that more uncertainty leads to softened price competition and increased firm profits, resulting in a welfare loss. This is consistent with the findings of the literature studying the role of information in competitive markets, where firms that learn too much may end up in a tough competition (Raith 1996, Villas-Boas 1994, Chen et al. 2001, Guo and Iyer 2010, Li 2006, Liu and Serfes 2002, Cabral and Villas-Boas 2005, Zhang 2011) and firms may choose not to learn as much as they could (Chen and Iyer 2002).

As we model the consumer voting process, our model is also related to the literature on group decision-making in political economy (Black 1948). Indeed, Hotelling's original model has been applied to election candidates by Downs (1957) leading to the Median Voter Theorem, arguing that strategic candidates will position themselves around the median voter. Our model is different from most of the literature as we do not allow candidates to be strategic individually, as they are either selected through a random process or by the firm. We mostly rely on plurality voting which is known to have inefficiencies when there are more than two candidates (Tullock 1959), therefore several other mechanisms have been suggested (Levin and Nalebuff 1995). Spatial models have been widely used for

election candidates (Stokes 1963) and several papers examine the difficulties of the multidimensional case (McKelvey and Wendell 1976, Calvert 1985). Although several other papers making important contributions to the theory of elections would be worth noting, we restrict our attention to a few recent papers that are directly related to parts of our analysis. These include Krishna and Morgan (2011) who examine voluntary versus compulsory voting, and (Castanheira 2011) who uncover the implications of voting for a losing candidate. In addition, recent interest in polls and market research has led to research on information aggregation in polls (Morgan and Stocken 2008) and the possibility of product flops due to non-truthful strategic answers in consumer surveys (Hummel et al. 2010).

3 Model

3.1 Consumer Preferences

We assume that consumers have heterogeneous tastes along a single dimension. A consumer's taste can be described by her location in the unit interval $x \in [0, 1]$. She has a sufficiently high valuation v for a product that perfectly fits her taste, but has a disutility from products that do not perfectly fit. Let r_i denote the location of product i in the $[0, 1]$ interval. We assume that consumers have a quadratic travel cost⁴, making consumer x 's valuation for product i

$$u_{xi} = v - t(x - r_i)^2. \quad (1)$$

We modify the typical linear city model to account for an important reason for soliciting consumer participation in product design: incomplete information about consumer preferences. We assume that a unit mass of consumers are either distributed uniformly in the $[0, 1/2]$ interval or uniformly in the $[1/2, 1]$ interval, calling the former left-sided demand and the latter right-sided demand. Firms do not know whether demand is left- or right-sided before choosing their location, but it is common knowledge that nature chooses the demand to be left-sided with probability $q^L = q \geq 0$, and to be right-sided with probability $q^R = 1 - q \leq 1$. Since the setup is symmetric around $1/2$ in q , we analyze the parameter range $0 \leq q \leq 1/2$ throughout the paper. In other words, demand is likely to be right-sided and q measures the level of uncertainty ranging from 0 to the maximal $q = 1/2$. This setup is similar to that by Meagher and Zauner (2004), but unlike them, we use a simple discrete distribution to be able to focus on consumer involvement rather than the nature of the uncertainty.

⁴Although Hotelling (1929) assumes a linear travel cost, a pure strategy equilibrium does not exist in the pricing stage when firms are located too close to each other. Therefore, the location choice game does not have a solution in pure strategies, but using the quadratic formulation suggested by D'Aspremont et al. (1979) ensures an equilibrium.

3.2 The Democratic Product Design Process

We refer to the process of consumer controlled product design, in which consumers participate in generating and voting about the final product, as a Democratic Product Design (DPD). We begin by assuming a relatively simple DPD process based on plurality voting, which we call $PL(k)$. We assume that consumers are non-strategic and they all participate in the voting process⁵. First, the firm solicits a k number of product ideas from random consumers, denoted by X_1, X_2, \dots, X_k . Each consumer will submit a product idea that exactly matches her preference⁶. That is, a consumer at location x will present a product idea that, if turned into a product, will be located exactly at x . After the candidates have been selected, all consumers vote for the k product ideas and the firm will create a product from the winning product idea. We assume that each consumer votes for the product candidate that is closest to her taste. The winner will be the candidate with the highest number of votes.

For example, let us assume that the demand is left-sided and a $PL(2)$ process is used. The firm solicits product ideas and randomly picks two candidates. The candidate closer to $1/4$ will win the vote. Since consumers are distributed symmetrically around $1/4$, the expected location of the winning product will be $1/4$ (or $3/4$ if the demand is right-sided). To determine the exact distribution, we first need to consider the two candidates: X_1, X_2 and determine which one is closer to $1/4$. Simple calculation⁷ shows that the distribution of the winning product location (X_w) is given by

$$\Pr(X_w < x) = F_w^L(x) = F_w(x) = \begin{cases} 8x^2 & \text{if } x \in [0, 1/4] \\ 8x(1-x) - 1 & \text{if } x \in [1/4, 1/2] \end{cases} \quad (2)$$

Not surprisingly, the expected location of the winning candidate is the middle of the interval $1/4$, but the process is not perfect and there can be a substantial amount of error in either direction. As k increases, a closed form derivation of the distribution becomes increasingly complex, but the numerical results depicted in Figure 1 paint a clear picture. With a larger k , more extreme candidates have a higher chance of winning, leading to a bimodal distribution when k is at least 4. As k approaches infinity the distribution of the winning candidate's location approaches the uniform distribution that is also the outcome of a $PL(1)$ process. Interestingly, the expected distance from the middle initially

⁵We do not need such a strong assumption: it is sufficient to assume that each consumer participates with the same positive probability, independently from her location.

⁶Although, we allow for any finite k number of candidates, firms realistically use only a few candidates (two to four) in most examples.

⁷The candidate closer to $1/4$ can be essentially determined as a maximum of two uniformly distributed variables, leading to the triangle distribution given in (2).

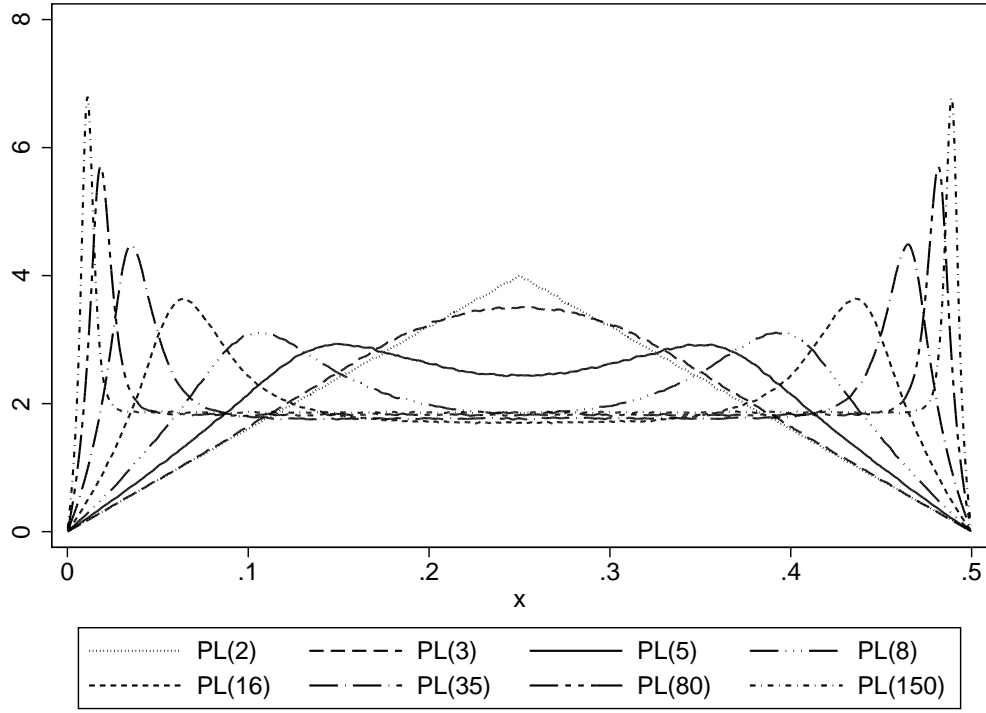


Figure 1: The p.d.f. of the winning location in a $PL(k)$ process.

increases with k as more extreme winners become more likely (see Figure 2).

Throughout the paper, we use the $PL(2)$ process as the primary example and calculate the exact equilibrium outcome for it. However, we will state our results in a more general form. To allow for more complex DPD processes, we employ a general setup, where we use the distribution of the winning product's location to describe the process. When the demand is left-sided, the winning product's location will be a random variable X_w with c.d.f. $F_w^L(\cdot) = F_w(\cdot)$ and p.d.f. $f_w^L(\cdot) = f_w(\cdot)$. Since consumers are symmetrically distributed in a half-unit interval, we assume that the distribution is symmetric with a support of $[0, 1/2]$, such that $F_w^L(x) = 1 - F_w^L(1/2 - x)$, yielding $f_w^L(x) = f_w^L(1/2 - x)$. When the demand is right-sided, the same c.d.f is shifted to the $[1/2, 1]$ interval such that $F_w^R(x) = F_w^L(x - 1/2)$, that is, $f_w^R(x) = f_w^L(x - 1/2)$. Furthermore, let σ_f denote the standard error of the above distribution.

3.3 Firms

In our general setup, we have two firms manufacturing their products at zero marginal cost that are identical except for the features that determine their location on the Hotelling line. As our goal is to

examine the market-level effects of democratic product design and compare it to traditional product design, we introduce a first stage in which firms simultaneously decide whether to engage in DPD (denoted by D), use traditional market research (M) or neither (N). Firms observe each other's choices of D , M or N and enter the next stage. In this second stage, firms engage in the location choice subgame that can take different forms depending on firm's choice of D , M , or N in the first stage: If Firm i decided to engage in DPD, the DPD process is implemented which determines its location. We assume that a firm that engages in DPD has to manufacture the exact product consumers voted for to avoid upsetting them. We study firms' incentives to deviate from the outcome of the DPD process at the end of Section 5. If a firm decided to use traditional market research (M) in the first stage, then it learns the nature of the demand perfectly and picks its product location accordingly. Finally, if Firm j decided not to use (N) either DPD or traditional market research, it simply picks a location based on its prior information (q)⁸. Once locations are determined and observed by both firms, they also learn whether demand is left- or right-sided. This is natural if at least one of them uses DPD, as observing the process reveals consumer preferences. Even if neither of them uses DPD, assuming that preferences are observed is consistent with the notion that unlike locations, prices are easily changeable. Firms can use different techniques to learn about demand after selling a limited number of units (Handel et al. 2012). Finally, after prices are set, consumers make their purchases and payoffs are realized.

4 Equilibrium Analysis

4.1 Monopoly

Let us first consider the case of a monopolist facing uncertain demand. A unit mass of consumers are uniformly distributed in $[0, 1/2]$ with probability q or in $[1/2, 1]$ with probability $1 - q$. We start with a benchmark case when the firm has to choose the location of its product without learning the exact nature of demand. In order to be able to compare with the duopoly case and to ensure tractability, we assume that v is high enough⁹ so that there is full market coverage and, for the most part, we only analyze the case of $q \leq 1/2$ w.l.o.g.

⁸Note that locations are determined simultaneously in the location choice subgame, that is, if Firm i uses DPD and Firm j does not, the outcome of the DPD process will not be observed by Firm j before choosing its location. The reason for this assumption is that traditional product design may take just as long as running a DPD process, in which case firms cannot wait for their competitor's DPD process to conclude to make a location choice. In some cases, it is plausible that running a DPD process takes much longer than standard location choice and Firm j observes the entire DPD process run by Firm i before making a choice.

⁹As we show in the proof the exact assumption is $v > 9t/16$.

Once the firm determines its location, it also learns the nature of the demand. If the firm picks location $r \geq 1/4$ and the demand is left-sided, then it will set a price of $p(r) = v - tr^2$, leading to a profit of $\pi^L(r) = v - tr^2$, whereas if $r \leq 1/4$ the profit will be $\pi^L(r) = v - t(1/2 - r)^2$. Similarly, if the demand is right-sided the profit will be $\pi^R(r) = v - t(1 - r)^2$, and $\pi^R(r) = v - t(1/2 - r)^2$ if $r \leq 3/4$ and $r \geq 3/4$ respectively. If the firm does not use DPD or market research and it has to rely on its beliefs about the nature of the demand, its expected profit will be $E\pi^N(r) = q\pi^L(r) + (1 - q)\pi^R(r)$. Note that this function is increasing for $r < 1/4$ and decreasing for $r > 3/4$, thus the firm will set a location in the $[1/4, 3/4]$ interval. Differentiating in this interval with respect to r yields that the optimal location and profits are

$$r^N = \begin{cases} 3/4 \\ 1 - q \\ 1/4 \end{cases} \quad \text{and} \quad \pi^N = \begin{cases} v - t(\frac{q}{2} + \frac{1}{16}) & \text{if } q \in [0, 1/4] \\ v - tq(1 - q) & \text{if } q \in [1/4, 3/4] \\ v - t(\frac{9}{16} - \frac{q}{2}) & \text{if } q \in [3/4, 1] \end{cases} \quad (3)$$

The above location choice shows how uncertainty about consumer preferences affects the firm's optimal product design. If the firm is certain enough that demand is left-sided, it will pick $r^N = 1/4$, whereas it picks $r^N = 3/4$ if the demand was likely to be right-sided. When demand is certainly left- or right-sided the firm will make a profit of $v - t/16$. If there is substantial uncertainty about the nature of the demand, the firm will pick an intermediate location. The expected profits will be the lowest when $q = 1/2$ and the firm picks $r^N = 1/2$ reaching a minimal payoff of $v - t/4$.

Let us now examine how using DPD affects prices and profits. Without loss of generality, let us assume that the demand is left-sided. Then, the DPD process leads to an outcome $r = X_w$ with c.d.f $F_w(\cdot)$. The firm will set a price such that even consumers in location 0 and $1/2$ buy the product. This will lead to a price of $p^D(r) = v - t(\max(r, 1/2 - r))^2$, leading to a profit of $\pi^D(r) = p^D(r)$. Note that the location X_w is a random variable, therefore we need to determine the expected profit $E\pi^D(X_w) = Ep^D(X_w) = v - tE(\max(X_w, 1/2 - X_w))^2$. Using the p.d.f. of the distribution of the winning product design, we get

$$\begin{aligned} E(\max(X_w, 1/2 - X_w))^2 &= \int_0^{1/2} (\max(x, 1/2 - x))^2 f_w(x) dx = \\ &= 2 \int_0^{1/4} (1/2 - x)^2 f_w(x) dx < (2/4) \int_0^{1/4} f_w(x) dx = 1/4. \end{aligned} \quad (4)$$

Therefore, we obtain that $E\pi^D(X_w) > v - t/4$, showing that when there is substantial uncertainty about consumer preferences, consumer DPD results in higher profits than direct location choice by the firm.

Finally, we consider the third case in which the firm uses traditional market research. After learning whether demand is left- or right-sided the location choice is trivial: the monopolist will choose the midpoint of the corresponding interval, resulting in a location of $1/4$ or $3/4$ and a profit of $\pi^M = v - \frac{t}{16}$. The following proposition summarizes the profit implications of the monopolist's choice.

Proposition 1 *When $0 < q \leq 1/2$, the monopolist's unique optimal choice is to use traditional market research. In particular:*

1. *The monopolist makes strictly higher profits using market research than either using DPD or not using any market research.*
2. *There exists a $0 < \bar{q} < 1/2$ such that the monopolist will make higher expected profits using DPD than not using any market research if and only if $\bar{q} < q \leq 1/2$.*

The proposition compares the choice between market research and DPD net of any cost advantages or direct benefits. The results show that using traditional market research is a dominant strategy for a monopolist. The results can be best understood in light of the basic trade-off between information acquisition and commitment. When there is sufficient amount of uncertainty about consumer preferences, the advantages of market research or DPD are apparent. Despite the uncertainty, the firm is able to sell a product that closely matches average consumer preferences, either by involving them in the design or using traditional market research. We call this a positive *learning* effect because as uncertainty about consumer preferences is revealed the firm can offer a product that fits consumer needs better, resulting in a higher profit. However, DPD has disadvantages as it does not yield an optimal product due to the imperfections in the process. The larger the potential error in the DPD process the farther the expected product location is from the ideal profit maximizing point. We call this a negative *commitment* effect because no matter how small the error is, the firm loses money by offering a suboptimal product that it committed to in the DPD. By using traditional market research, the monopolist can avoid this negative effect and learn consumer demand without any commitment to a potentially suboptimal product. Therefore, using market research is a strictly dominant strategy even though DPD has the same learning value.

However, DPD and market research may involve different costs. For example if DPD costs significantly less, the monopolist may find it optimal to use DPD. The second point of the proposition shows that this is true if and only if there is sufficient uncertainty about consumer preferences. The intuition

follows clearly from the trade-off identified above as the positive information effect will outweigh the negative commitment effect only if the uncertainty is relatively high. Also, market research typically does not provide perfect information in a realistic setting. Depending on the amount spent on market research might provide worse information than DPD. In this case, the monopolist would have to trade off the better information gained from DPD for the necessary commitment. Furthermore, as we elaborate in Section 7, consumers may directly benefit from DPD if they enjoy the involvement in the product design process. This might also drive the monopolist to use DPD instead of traditional market research.

Since a monopolist may have various reasons to use DPD as explained above, it is useful to examine how the profits depend on the exact DPD process employed. As we determined previously, firm profits under DPD are $E\pi^D = v - 2t \int_0^{1/4} (1/2 - x)^2 f_w(x) dx$ in the case of a general DPD outcome distribution. When the DPD process is a simple DPD process of a majority choice between $k = 2$ candidates, that is, when $f_w(x) = f_w^{(2)}(x) = 16x$ for $0 \leq x \leq 1/4$, the exact solution is $E\pi^D(X_w) = v - (11/96)t$. If the monopolist uses more than two candidates, the profits initially decrease as k increases, because extreme candidates have a higher likelihood of winning the vote (see Figure 2). In fact, when k is 15 or more, the monopolist's profits are lower than in the case of simply picking a random consumer's ideal product. In summary, if the monopolist chooses to utilize DPD, it can maximize profits by using only two candidates in the process.

4.2 Duopoly

We continue the analysis by examining the duopoly case, where firms first simultaneously decide whether to use DPD, traditional market research or neither, then determine their locations, and finally set prices. Using backward induction, we first consider the nine possible location choice subgames after the DPD/market research decision has been made. We divide these cases into three categories based on whether firms use DPD: i) neither firm engages in DPD and they choose their location, ii) one firm engages in DPD while the other chooses its location, iii) both firms use DPD, thereby determining their locations.

Firms do not use DPD. In the subgames where firms do not use DPD, they both have a location decisions to make. They can either conduct traditional market research to aid this decision or rely only on their prior information about demand. For example, if both firms use market research and they find out that the demand is right-sided, the location choice game is equivalent to the classic

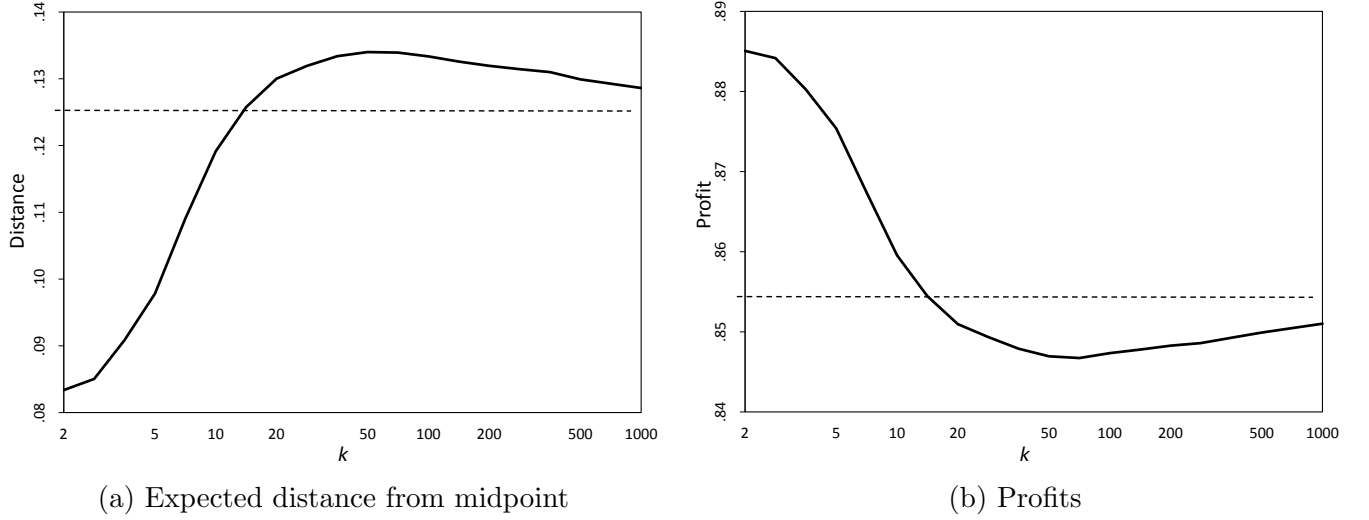


Figure 2: The expected distance of the winning candidate from the midpoint in a $PL(k)$ process and monopoly profits when using it ($v = t = 1$). The dashed line depicts the baseline value obtained when using a simple uniform sampling (equivalent to a $PL(1)$ process).

linear city model (with the exception that consumers are distributed in $[1/2, 1]$ instead of the typical $[0, 1]$). As D'Aspremont et al. (1979) show, the principle of maximum differentiation applies and firms locate far from each other. One caveat of this principle is that in linear city models firms are typically not allowed outside the interval where consumers are located, therefore they locate at the endpoints. However, firms have an incentive to move even further out: the equilibrium locations are $(-1/4, 5/4)$ when consumers are located in the $[0, 1]$ interval. Since our model assumes uncertainty about consumer locations, we allow firms to locate anywhere on the real line. In case of a certain right-sided demand this translates to locations $(3/8, 9/8)$ and to profits of $\pi_i^{MM} = \pi_j^{MM} = 3t/16$

If firms do not learn about consumer preferences through market research, their locations will be more dispersed. For example, when $q = 1/2$, they will locate at $(1/12, 11/12)$. In general, as priors about the location of the demand become more diffused, firms will locate farther apart from each other to cover all possible consumer locations:

Lemma 1 *When neither firm uses DPD or market research, firms' locations and profits are*

$$r_i^{NN}(q) = \frac{4q^2 - 16q + 9}{24}, \quad r_j^{NN}(q) = \frac{27 - 4q^2 - 8q}{24}, \quad \pi_i^{NN}(q) = \pi_j^{NN}(q) = \frac{t(4q^2 - 4q - 9)^2}{432}.$$

Note that firms make lower profits when they both use market research than when they do not

($3t/16$ compared to $25t/108$, when $q = 1/2$). This result is consistent with a stream of literature arguing that more information about the demand can hurt firms by leading to tougher price competition (Raith 1996). However, as we show in the proof of Proposition 2, firms have an incentive to use market research when their competitor does not, therefore an equilibrium with neither firm using neither DPD nor market research is not sustainable.

Only one firms uses DPD. Moving on to the second category, we assume that only Firm 1 engages in DPD. Firm 2 either uses traditional market research or relies only on its priors to determine its location. We start with the case when Firm 2 does not use market research and determine its best response (in location choice) to Firm 1's choice of DPD. Note that Firm 2 does not observe the outcome of Firm 1's DPD process, but it knows the distribution of potential outcomes. The p.d.f. of this distribution is

$$f(x) = \begin{cases} q \cdot f_w(x) & \text{if } x \in [0, 1/2] \\ (1 - q) \cdot f_w(x - 1/2) & \text{if } x \in [1/2, 1] \end{cases}, \quad (5)$$

where $f_w()$ is the p.d.f. of the DPD process in the case of a left-sided demand and σ_f is its standard error. Maximizing the expected profit of Firm 2 along its possible location choices, we get the best response.

Lemma 2 *There exists a $\bar{\sigma} > 0$ such that if $\sigma_f < \bar{\sigma}$ and $0 < q < 1/2$ then Firm 2's unique best response to Firm 1's choice of using DPD is to locate at*

$$r_2(q) = \frac{7}{4} - \frac{q + \sqrt{q^2 - q + 1 + \frac{4\sigma_f^2}{3}}}{2} > 1.$$

We obtain that if the DPD process is relatively precise, resulting in a product close to the middle of the interval where consumers are located, the best response is to pick a far right location that is outside the range of possible consumer locations. This is a unique best response as long as $0 < q < 1/2$. Naturally, when $q = 0$ and the demand is certainly right-sided Firm 2 will be indifferent between picking $1 + y$ and $1/2 - y$ for any y and when $q = 1/2$, Firm 2 will be indifferent between $1 + y$ and $-y$. However, when demand is likely to be right-sided, but there is a small chance of it being left-sided, Firm 2 wants to locate relatively far from the $[1/2, 1]$, where Firm 1 will be located around the center ($3/4$) with a high probability, but also wants to avoid $[0, 1/2]$ where Firm 1 can show up with a positive probability (see Figure 3).

The combination of these two manifestations of the principle of maximum differentiation results in a location $r_2 > 1$ when demand is more likely to be right sided.¹⁰ Note that these results hold when

¹⁰When demand is more likely to be left-sided Firm 2 will locate left of 0.

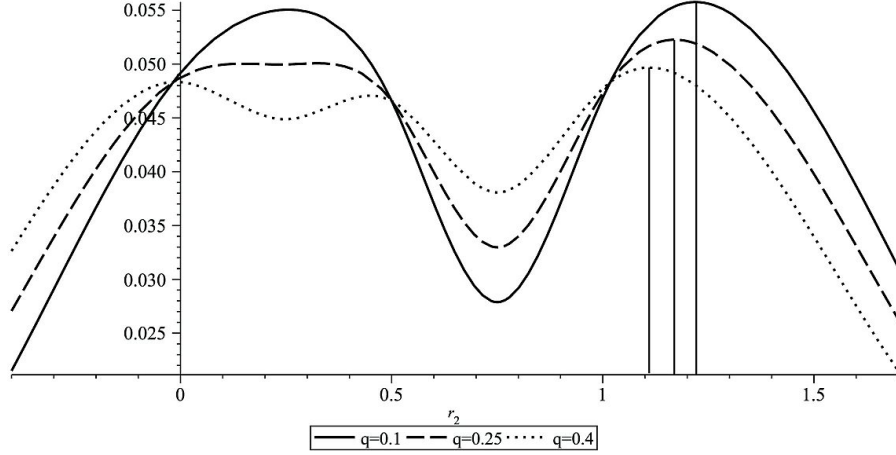


Figure 3: Firm 2's profit as a function of its location r_2 , when Firm 1 uses a $PL(2)$ DPD. The three curves show profit function for three levels of uncertainty and the vertical lines point out the best response.

the DPD process is relatively centered with a low variance. All of the $PL(k)$ processes satisfy this condition and result in the above mentioned extreme best response.

In essence, when Firm 1 uses DPD and Firm 2 does not use any market research, Firm 1 ends up in a central location and Firm 2's best response is outside of this central zone. The outcome is similar when Firm 2 uses market research (while Firm 1 still uses DPD). If, for example, demand is right-sided Firm 2 can now observe this and give a slightly better response. In fact, the best response will be symmetric around $[1/2, 1]$, but, as we show in the proof of Proposition 2, this case is equivalent to the above analysis with $q = 0$. Hence, Firm 1 again ends up in a central location with Firm 2 pushed out to the side.

Both firms use DPD. In the third case, where both firms engage in DPD, neither of them has to make a location choice. Their location is simply determined by the DPD processes. The locations will be given by two random draws from the distribution with p.d.f. $f_w^L(\cdot)$ or $f_w^R(\cdot)$ (depending on whether demand is left- or right sided), bringing the two firms substantially closer than in the first subgame and likely closer than in the second subgame. Note that the level of uncertainty (q) does not affect the expected payoff as the DPD process will ensure a product that is mostly centered in the consumer preference interval no matter if demand is left- or right-sided.

Use of DPD in equilibrium. Combining the above explored subgames allows us to determine the equilibrium of the first stage and see when firms engage in DPD. We examine both the pure- and mixed-strategies in the first stage where firms decide whether to use DPD. We obtain the following

results. Recall that σ_f denotes the standard error of the DPD process defined by $f(\cdot)$.

Proposition 2

1. *There exists a $\bar{\sigma}' > 0$ such that if $\sigma_f < \bar{\sigma}'$ then the unique pure-strategy equilibrium is asymmetric with one firm using traditional market research and the other engaging in DPD.*
2. *If $\sigma_f < \bar{\sigma}'$, there is a unique symmetric equilibrium in mixed first-stage strategies in which firms use DPD with a positive probability $0 < P < 1$ and use market research with probability $1 - P$.*
3. *The $PL(2)$ process satisfies $\sigma_{PL(2)} < \bar{\sigma}'$ and $P = \frac{175}{226} \cdot \frac{4176 - 311\sqrt{146}}{3237 - 245\sqrt{146}} \approx 0.494$.*

The proposition demonstrates that if DPD tends to result in a product that is not too far from average consumer preferences then only one firm engages in DPD, whereas its competitor uses traditional market research. The intuition can be best described relying on the learning and commitment components of DPD and market research identified in the monopoly analysis. The positive learning effect of market research and DPD are the same as these activities provide a perfect signal about the nature of the demand. Using either of these gives a firm an advantage if its competitor is not perfectly informed and vice versa. Thus, as long as DPD and market research are not too costly, not using either of them is a dominated strategy.

The choice between DPD and market research is driven by the commitment effect. When using DPD, a firm commits to the product design that consumers vote for. This product design is likely to be close to the average preferences, not at an extreme location as we would expect under standard product differentiation. That is, by using DPD, a firm can commit to not consider differentiation. The best its competitor can do, if not using DPD, is select an extremely differentiated location. Hence, firms have a clear incentive to use DPD if their competitor does not. On the other hand, firms do not want to copy their competitors by using DPD as profits suffer tremendously if firms locate close to each other resulting in intense price competition.

The mixed strategy equilibrium directly corresponds to the above described asymmetric equilibrium. When firms cannot coordinate in who will engage in DPD, they will both mix and choose DPD with the given probability and market research otherwise. It is interesting to note that the choice of DPD preempts the coordination problem encountered when both firms have to pick their locations in the second stage. Although the locations are fully determined in equilibrium, firms have to decide who will be on the left and who will be on the right. Usually firms have a history of being on one or the

other side, but if there is no such coordination device, the choice of DPD can dissolve this coordination problem. In the mixed strategy equilibrium it is still possible that firms both have to choose locations, but the probability is fairly low.

It is useful to compare the profits firms can expect in different scenarios. When considering the asymmetric equilibrium - which is clearly beneficial for the firm using DPD ex post - we can calculate the profit a firm can expect ex ante when randomly assigned to a role as half of the total industry profits. These are strictly lower than if both firms used traditional market research or neither of them used market research. The expected profits are reduced even more in the symmetric mixed strategy equilibrium where both firms might use DPD at the same time. Firm profits are thus hurt by the use of DPD due to the strategic effects of commitment. On the other hand, consumer surplus increases as a result of DPD as consumers are better off both in the asymmetric equilibrium and the symmetric mixed equilibrium. The increased consumer surplus is not only a result of products in the market that fit the average consumer's taste better, but also a result of increased price competition.

5 Deviation from Consumer Votes

An important question regarding democratic product design is whether firms have an incentive to follow consumer votes or potentially deviate from them. In practice, firms have various options to manipulate the results in more or less obvious ways, but consumers may be upset if the final product is different from what they voted for and this negative sentiment could translate into a decreased valuation for the product. Although most examples of democratic product design show that firms follow the vote results, there is anecdotal evidence that consumers do penalize firms for breaking their promises in general. Indeed, an Accenture study¹¹ reports, with respect to marketing and sales practices, that 59% of consumers finds it “extremely frustrating” if firms “promise one thing but deliver another”. Another study¹² found that 38% of respondents switched to another product soon after experiencing a broken promise and 52% considered switching.

In this section, we incorporate the decreased valuation from a potentially broken promise. As opposed to the basic model, where we assume that firms that choose to engage in DPD keep their

¹¹ “2012 Global Consumer Pulse Research”, Accenture, 2012 - available at <http://www.accenture.com/SiteCollectionDocuments/PDF/Accenture-Global-Consumer-Pulse-Research-Study-2012-Key-Findings.pdf> (accessed August 12, 2014)

¹² “Promises, Promises: Easily Made, Easily Broken”, Accenture, 2012 - available at <http://www.accenture.com/SiteCollectionDocuments/PDF/Accenture-Broken-Promises-Survey-Retain-Customers.pdf>

promise to consumers and implement the product winning the consumer vote, we now formally model how firms are penalized for deviating from the vote outcome. We allow firms to select a location different from the DPD outcome in the second stage of the game. Consumers then observe both the DPD outcome and the product's final location which, if different from the winner in the vote, reduces their valuation. We modify equation (1) to include this reduction in utility when Firm i uses DPD:

$$u_{xi} = v - t(x - r_i)^2 - D(g_i, x, r_i, X_{wi}) \quad (6)$$

where $D(g_i, x, X_{wi})$ is the disutility experienced by consumer at location x when the product offered by Firm i is located at r_i instead of the vote outcome of X_{wi} . The parameter g_i measures how sensitive consumers are to Firm i 's deviations from the DPD outcome. There are various different possible formulations for the consumer disutility. Here, we present an analytically well tractable functional form, where the disutility increases with the distance the final product has moved from the vote outcome, but making sure that consumers do not value a product less if it came closer to them. Formally, we set

$$D(g_i, x, r_i, X_{wi}) = \min \{g_i t |r_i - X_{wi}|, t(x - r_i)^2 - t(x - X_{wi})^2\} \quad (7)$$

The first term simply measures how far the final product is from the vote outcome, whereas the second term ensures that the disutility cannot exceed the utility improvement from the reduced travel costs. Therefore, consumers will never value a product lower if it is closer to them than a more distant vote outcome. However, they certainly do penalize the firm when the product moves farther away than the vote outcome. We use g_i to measure how much the distance affects the disutility and the t multiplier to rescale to the same unit as the travel costs. Consequently, when $g_i = 0$, Firm i is free to pick any location regardless of the vote outcome without worrying about consumer backlash, making this case equivalent to using traditional market research. On the other hand, a sufficiently high g_i captures a situation where consumers penalize the firm for deviating from the vote outcome.

In the remainder of this section, we first examine the case of $g_i = g_j = g$ and analyze how the equilibrium of the model changes with g . Then, we endogenize the value of g_i by allowing firms to make stronger or weaker promises about sticking with consumer choice. To simplify the analysis, we assume that the winning product's distribution is sufficiently centered (σ_f is small).

5.1 Use of DPD in equilibrium

The penalty for deviating from the DPD outcome does not substantially affect a monopolist's choice. The optimal strategy is to use market research regardless of consumer sensitivity. The case of a duopoly is more complex. Similarly to the analysis of the main model, we first consider the location subgames after each firm decided whether to use DPD, market research, or neither.

In the case where both firms use DPD, they will be able to profitably move away from each other and the center when g is low enough. As g approaches 0, they will be differentiated to the same extent as if they both used market research. When only one firm uses DPD, we observe a similar trend, but the consequences are different. As g decreases, the firm using DPD moves out from its dominant central position, whereas its competitor can move closer to the center (still keeping a substantial distance). In this case, lowering g reduces the asymmetry in profits, hurting the firm using DPD, but helping its competitor.

Combining the different subgames, we determine the DPD choice equilibrium for different values of $g > 0$ below. We naturally expect a high g to replicate the results of Proposition 2, whereas $g = 0$ implies that firms are indifferent between DPD and market research since, in effect, these two are equivalent.

Proposition 3 *If σ_f is small enough, there exist $0 < g_A < g_B < g_C$ such that the equilibria in pure strategies have the following properties.*

1. *If $0 < g < g_A$, then both firms use traditional market research in the unique equilibrium.*
2. *If $g_B < g < g_C$, then both firms use DPD in the unique equilibrium.*
3. *If $g_A < g < g_B$ or if $g_C < g$, then one firm uses DPD while the other employs market research in the unique equilibrium.*

The results are consistent with our intuition high values of g , but reveal an interesting pattern for intermediate values. When consumers are moderately sensitive to deviations from the vote outcome, both firms use DPD in equilibrium. The reason is that they can maintain a reasonable level of differentiation by moving away from each other. The asymmetric equilibrium is not sustainable, because the firm using market research, forced far out of the market, now has an incentive to be offensive and also use DPD. This results in a prisoner's dilemma, as firms make strictly lower profits when they both use DPD, than when they use market research. For a small range of g values we again

obtain the asymmetric equilibrium. Finally, when consumers are only slightly sensitive to deviations and g is low, both firms use traditional market research. The reason is that, the asymmetric setting (one using DPD, the other market research) is unfavorable to the firm using DPD. It restricts its strategy space, but does not provide enough commitment power to force its competitor to the edge of the market. The above pattern of results nicely illustrates how the role of DPD changes as the strength of the commitment to the vote outcome diminishes.

5.2 Self-imposed penalty for deviating

The extent to which consumers are sensitive to deviations from the outcome of a potential vote can often be influenced by the firm engaging in DPD. For example, increasing the transparency of the process makes it easier for consumers to follow the vote. Making widely broadcasted promises upfront also leads consumers to expect the firm to stick with the vote outcome, whereas warning consumers that the vote outcome may not be implemented (possibly because of feasibility constraints) can neutralize the negative sentiments.

We therefore endogenize the choice of g_i and allow firms to determine the level of penalty for deviating from the vote outcome. To focus on the strategic effects, we assume that there is no direct cost or benefit to any particular choice of g_i value and to maintain analytical tractability, we assume that firms can choose between two discrete levels of g_i . Firms can either set $g_i = 0$ or $g_i = \bar{g} > 0$. We show that if \bar{g} is large enough, then at least one firm sets $g_i = \bar{g}$ in equilibrium.

Corollary 1 *If σ_f is small enough, then using the thresholds $0 < g_A < g_B < g_C$ from Proposition 3, we get the following equilibria.*

1. *If $0 < \bar{g} < g_A$, there is no equilibrium where a firm sets $g_i = \bar{g}$ and then uses DPD.*
2. *If $g_B < \bar{g} < g_C$, then both firms set $g_1 = g_2 = \bar{g}$ and they both use DPD in the unique equilibrium.*
3. *If $g_A < \bar{g} < g_B$ or if $g_C < \bar{g}$, then in any equilibrium there is exactly one firm setting $g_i = \bar{g}$ and using DPD.*

The intuition follows from the previous results. When \bar{g} is low, it is not possible to commit to the DPD outcome and firms will just use market research. As \bar{g} increases, it becomes possible to commit to DPD to some extent. When $g_B < \bar{g} < g_C$, firms will end up in a prisoner's dilemma: both of them will set $g_1 = g_2 = \bar{g}$ and both will engage in DPD. They are able to differentiate somewhat, but will

be located relatively close to the vote outcomes that are themselves close to each other. Finally, when a very strong commitment to the DPD outcome is possible, such that $g_C < \bar{g}$ (or $g_A < \bar{g} < g_B$), there will be multiple equilibria, where one or both firms set $g_i = \bar{g}$. In the DPD subgame one firm will always use DPD and not deviate from the vote results, whereas the other firm will freely position its product either by using traditional market research right away or using DPD, but deviating because of setting its g_i to 0. Eventually, firms will end up at the same locations as in the asymmetric equilibrium of our main model. Hence, this case essentially replicates our main results.

In summary, we show that if there is a tool for firms to strongly self-commit then at least one of them will always do. If the self-commitment tool is very effective, then exactly one firm will commit to and engage in DPD, whereas if it is moderately effective, both firms will commit to and engage in DPD. In the latter case, firms cannot resist the potential advantage of self-committing, but by both of them doing it, they end up in a price war with reduced profits. When the self-commitment tool is not effective no firm will commit to and go through with DPD.

6 Manipulating the Vote by Preselecting Candidates

As we have shown so far, firms employing DPD generally have an incentive to deviate from the vote outcome in the absence of a commitment device. When consumers do not punish firms for such deviations, DPD becomes very similar to traditional market research. In this section we study how firms can have more control over the DPD outcome even if they commit to the formal vote results in fear of a consumer backlash.

Firms often solicit consumers for their ideas and put these up for a vote (MyStarbucksIdea, Lego, Burberry). In fact, most crowdsourcing platforms rely on ideas submitted directly by consumers. But instead of soliciting candidates for a majority vote from random consumers, the firm can pick candidates itself and have consumers vote on them. Sometimes firms admittedly generate the candidates themselves, in other examples, even though the process is fairly transparent, firms can employ very long campaigns where they can have a good amount of flexibility in determining the final candidates. For example, the Dewocracy campaign lasted for sixteen months and initially started with seventeen candidates that were later narrowed down to three in the final where most consumers participated. In such a long and complex campaign firms have the opportunity to introduce their own candidates without upsetting consumers. But even in cases where firms solicit original consumer ideas, it is possible that the firm has some influence. Starbucks' top winning consumer idea, "Splash Sticks" (supposedly

selected from 80,000 consumer ideas) turned out to be a product feature that was already used by the company in Japan.

To give more control to firms over the final outcome in our model, we slightly modify the $PL(k)$ process. We denote this process by $PREPL(k)$: a firm using DPD selects k candidates that do not have to coincide with any consumer's taste. The winner is decided by a majority vote where consumers are not strategic and each of them votes for her favorite product. If consumers are indifferent between two products they pick one randomly and, in case of a tie for the top design, the firm picks the winner.¹³ We start by showing that a firm can achieve essentially any location combination $r^L < r^R$ if it can pre-select a high enough number of candidates.

Lemma 3 *If $r^L < r^R$ and $0 < r^L + r^R < 2$, then there exists a \underline{k} such that a firm can ensure that the outcome of the DPD process is r^L in case of a left-sided demand and r^R in case of a right-sided demand by pre-selecting the candidates in a $PREPL(k)$ process with a $k \geq \underline{k}$.*

Thus, by choosing enough candidates firms can achieve complete freedom in location choice and only take advantage of the informational effect of DPD by learning whether demand is left- or right-sided.

Therefore, if we modify by the basic model presented in Section 3 to use the $PREPL(k)$ process, firms will be indifferent between using marketing research and DPD as long as k is high enough. Below, we determine the minimum number of candidates needed to achieve this.

Corollary 2 *Firms can achieve the equivalent of traditional market research using DPD and $PREPL(k)$ iff $k \geq 4$.*

In essence, using four preselected candidates or more gives firms complete freedom in their location choice even if they fully commit to the vote outcome. Although in our model firms are then indifferent between DPD and market research they may have a preference for one or the other depending on the costs. Also, as we examine in Section 7, DPD potentially has other benefits. Preselecting the candidates allows firms to take advantage of these without braking any promises.

One caveat of the above results is that firms need to use a number of identical candidates that consumers might find odd. Also, some candidates may be outside of the area where consumers are located, leading to candidates that are obviously not desired by any consumer. Imposing some restrictions to make the setting more realistic gives similar results where firms can achieve outcomes similar

¹³This assumption is not critical, but makes the analysis cleaner.

to as if they used market research. We present one such more realistic result where firms are restricted to two candidates.

We solve our full basic model presented in Section 3 with modification that any firm using DPD can use the *PREPL*(2) DPD process.

Corollary 3 *When firms are restricted to a PREPL(2) DPD process, they both decide to use DPD in equilibrium. Firm locations will be either*

$$r_i^L = 0, r_j^L = 1/2, r_i^R = 1/2, r_j^R = 1, \text{ or}$$

$$r_i^L = -1/6, r_j^L = 1/2, r_i^R = 7/6, r_j^R = 1/2.$$

In essence, even if firms can only use two pre-selected candidates for a majority vote, they will both engage in DPD. Since firms are able to differentiate less, they make lower profits than if they both used traditional market research, resulting in a prisoner's dilemma. We have thus shown that although it might be tempting for firms to exercise more control over the final product, this may be a trap leading to intense price competition and lower profits.

7 Consumer Engagement in DPD

A common argument for DPD is that it directly creates value by increasing the utility that consumers get from the product. In other words, consumers are willing to pay more for a product if it was created with their involvement than for the exact same product designed solely by the firm. Fuchs et al. (2010) show that consumers who are empowered to vote for a product exhibit an increased willingness to pay for it, but this benefit diminishes if the outcome does not reflect their preferences. In this section we examine both the positive and potentially negative effects that engagement may have on consumer valuation. We first study how the equilibrium changes if consumers that participate in DPD have an increased valuation for the winning product. Then we consider the potential disutility consumers receive from voting for a product design that is not supported by the majority.

7.1 Increased valuation from engagement

To account for the potential direct, valuation-increasing effect of DPD, we modify our basic model and incorporate an extra term in the consumer utility formulation of (1). We begin our analysis by assuming that consumer x 's valuation for product j is

$$u_{xi} = v_b + v_{di} - t(x - r_i)^2, \tag{8}$$

where the utility derived from an ideally designed product $v = v_b + v_{di}$ now includes a base component and a DPD component. The extra value derived from DPD is $v_{di} = v_d > 0$ if Firm i engages in DPD and $v_{di} = 0$ otherwise. Aside from this modification the model setup remains the same. Our goal is to investigate how the results change as v_d increases from 0.

We first look at a monopolist along the same lines as in Section 4.1. After finding out whether demand is left- or right-sided, the monopolist using DPD sets a price so that even consumers at the extremes buy its product. For a left-sided demand and a location r this results in a profit $v - t(\max(r, 1/2 - r))^2$. This is clearly increasing in v and using $v = v_b + v_d$ it is also increasing in v_d . Hence, for any given DPD outcome the monopolist will make higher profits if v_d increases. The results are the same for a right-sided demand, demonstrating that regardless of the DPD process used, a monopolist always benefits from increased consumer valuation of engagement in the DPD process. As a consequence, if v_d is high enough the monopolist will find it profitable to use DPD instead of relying on traditional market research.

Moving on to the case of a duopoly, we note that in horizontal differentiation models an increased consumer valuation for both firms does not change the equilibrium as competition prevents firms from increasing prices enough for valuations to be binding (as long as valuations are high). Therefore, we do not expect any change in the subgames where both or neither firms use DPD. However, an asymmetry is created in valuations in the subgame where only one firm uses DPD. The increased valuation gives an advantage to the firm that uses DPD and its profits will be increasing in v_d , whereas its competitor's profits will be decreasing. As we show below, above a certain threshold this asymmetry becomes significant and results in a different equilibrium.

Proposition 4 *There exists a $\underline{v_d} > 0$, such that, if $v_d > \underline{v_d}$ then both firms use DPD. Equilibrium profits will be strictly lower than if neither firm used DPD regardless of the DPD process used.*

The result highlights an interesting strategic aspect of DPD. As the value of consumer engagement in the DPD process increases, firms are more likely to engage in DPD which leads to a reduction in profits. The intuition is as follows: the direct increase in consumer valuation makes it attractive for firms to engage in DPD and makes it especially unattractive not to engage in it, when the other firm does so. In other words, as consumers value the engagement in DPD more, the asymmetric outcome becomes less favorable for a firm that does not use DPD. Not only can its competitor better position its product covering a large percentage of consumer preferences, but DPD can also provide extra

benefits to the customers. Unlike in the benchmark case of $v_d = 0$, when the firm not using DPD in the asymmetric case was able to make reasonable profits by positioning itself at one extreme, DPD is a dominant strategy when v_d is high enough. Compared to the mixed strategy equilibrium of the basic model, we show that profits are lower.

Corollary 4 *If $v_d > \underline{v_d}$ and a $PL(2)$ process is used, profits will be strictly lower than the expected profits in the mixed strategy equilibrium described in Proposition 2 for the case of $v_d = 0$.*

This leads us to the conclusion that a seemingly beneficial increased engagement value of DPD can be dangerous for firms. Similar to a Bertrand supertrap, an increase in v_d leads to a higher likelihood of imitation of the decision to engage in DPD. Eventually, this results in reduced profits as both firms start using DPD inducing intense price competition. Thus, firms should be careful in increasing consumer engagement in their DPD process as even a unilateral increase in consumer benefits could trigger a response by competitors to use DPD and increase engagement on their side.

7.2 Voting for a loser

DPD certainly requires more or less consumer involvement and we have shown how the positive effects of engagement change firms' strategies. However, in a majority vote not all voters are necessarily happy with the results. We already capture the first order disutility consumers face when the chosen product is not perfect by employing the typical horizontal travel cost formulation. In some cases this may not be enough. For a given customer and given design outcome, it is plausible that consumers like the product more if they voted for it than if they did not. In the beginning of this section we introduced v_d to measure the extra utility consumers get from engaging in the DPD process. Below, we examine what happens when consumers' valuation of a product is affected by whether they voted for it or not. Therefore, we redefine v_{di} as

$$v_{di} = v_{dix} = \begin{cases} 0 & \text{if firm } i \text{ does not use DPD} \\ v_d & \text{if consumer } x \text{ voted for the winning product candidate of firm } i \\ v_d - v_n & \text{if consumer } x \text{ voted for a non-winning product candidate of firm } i \end{cases}$$

Our previous setup is a special case of this more general setting with $v_n = 0$. Here $v_n > 0$ measures the negative utility consumers get from voting for a product candidate that does not win the vote. As before, we assume that consumers are non-strategic and simply vote for the product that is closest to their location.

We first examine the monopoly case and for tractability, we focus on a $PL(2)$ process. When there are two randomly selected candidates, it is fairly easy to determine the distribution of the set of voters who voted for the winner. Let us assume that the demand is left-sided and consumers are uniformly distributed in $[0, 1/2]$. We provided the distribution of the location of the winning candidate in (2). Here, we determine who could have voted for the winning candidate given its location. If $X_w = r \leq 1/4$, then there are two cases: the competing, non-winning candidate (X_n) was positioned either to the left or the right of X_w . If $X_n < r$ then it could have been anywhere between 0 and X_w . However, if $X_n > r$ it had to be between $1/2 - r$ and $1/2$. Therefore, the distribution of $(X_n | X_w = r)$ is uniform in $[0, r] \cup [1/2 - r, 1/2]$. Hence the set of consumers who did not vote for the winning candidate is $[0, \hat{X}]$ with $1/2$ probability and $[1/2 - \hat{X}, 1]$ with $1/2$ probability where \hat{X} is distributed uniformly between $r/2$ and r . More importantly, for the monopoly case, the extreme consumer on the left voted for the winner with exactly $1/2$ probability conditional on any $X_w = r$ location. We now examine how a monopolist is affected by the differentiated consumer valuations.

Lemma 4 *If the monopolist uses a $PL(2)$ process, then its expected profit is*

$$E\pi^D = \begin{cases} v_b + v_d - \frac{2}{3} \frac{v_n^2(3t-4v_n)}{t^2} - \frac{11t}{96} & \text{if } v_n \leq t/4 \\ v_b + v_d - \frac{v_n}{2} - \frac{7t}{96} & \text{if } v_n > t/4 \end{cases}$$

Apparently, the monopolist's expected profit is decreasing in v_n , that is, a negative sentiment about not voting for the winning product always hurts the monopolist. The derivative ranges from 0 to $-1/2$ as v_n increases, while the derivative with respect to v_d is $+1$, showing that although the negative effects of consumer engagement cannot be ignored, they are outweighed by an increase of the same magnitude in the positive effects. In other words, consumer excitement about the voting process will only reduce profits if consumers who did not vote for the winning product are significantly more upset (in this model twice as much) than how satisfied those are who voted for the winning product.

We now analyze how profits are affected by a change in $v_n > 0$ in the case of duopoly. As we have seen above a high enough v_d makes both firms use DPD, thus, we focus on this particular subgame. We assume that both firms use a $PL(2)$ DPD process and determine how their profits change as a function of v_n . For simplicity we restrict our attention to values of v_n that are not too high. This way, we are able to avoid discontinuities in the profit functions in the relevant intervals. We expect the same results to hold for larger values, but the analysis becomes more tedious.

Proposition 5 *There exists a $\bar{v}_n > 0$ such that if $v_n < \bar{v}_n$ then expected equilibrium prices and profits are increasing in v_n .*

Surprisingly, a negative sentiment about the winning product can increase firm profits in a duopoly setting. The intuition is rather intriguing. Depending on the outcome of the two DPD processes there are cases where the negative sentiment is irrelevant because the marginal consumers (those who are close to being indifferent between the two products) either voted for both or neither of the two firms' winning products. In these cases prices and profits are not affected by an increase in v_n . However, there are potential outcomes where a negative sentiment can shift the location of the indifferent customers, as consumers who are otherwise indifferent voted for one but not the other winning product. This gives an advantage to the firm whose winning product the indifferent customers voted for, but since our setup is symmetric the exact opposite situation happened with the same probability. Pairing and integrating over these asymmetric outcomes we have to add up pairs where one firm had a valuation advantage. The sum of profits will be higher than in the case of no valuation difference because the asymmetry induces extra differentiation. In other words, firms benefit from more negative sentiments because in the cases when these sentiments differentiate between firms, price competition is softened, leading to higher profits. Thus, negative sentiments create extra differentiation with a positive probability leading to higher profits. In essence, consumers can spontaneously reinforce the differentiation initiated by firms.

In summary, this section demonstrates how consumer engagement affects firms using DPD. As one would expect, positive engagement makes DPD a more attractive option for the monopolist. If consumer product valuations increase enough in a DPD process, the monopolist will prefer DPD over traditional market research. A negative engagement hurts a monopolist, although to a lesser extent. Surprisingly, these results reverse in duopoly due to the strategic effects of DPD. An increased positive engagement value for consumers gives an incentive for firms to imitate their competitors by using DPD. This intensifies price competition due to a smaller distance between firms. However, a negative sentiment about a winning product design that the consumer did not vote for can create extra differentiation between firms, leading to higher prices and profits.

An important implication of these results is that firms should be considerate of the level and nature of consumer engagement they wish to achieve as they can find themselves in a trap with intense price competition. They should also pay extra attention to how they run their "election campaigns" as negative sentiments from the losers can negatively affect profits in a monopoly. On the other hand, in a competitive environment negative sentiments can serve as a tool to differentiate and alienating consumer who will buy a competitor's product anyway is not necessarily detrimental. For example, if

avid Coke fans are upset by the fact that a Coke-like candidate did not win Dewmocracy may actually help Mountain Dew differentiate its new product.

8 Conclusion

Voting on products and features is an increasingly popular tool among firms to increase consumer participation in product design. In this paper we study the strategic implications of using a democratic product design process. Our main results show that the commitment to the results of consumers' votes has a crucial impact on firm strategies. Even when uncertainty is low and firms cannot learn much about consumer preferences, using consumer votes serves as a commitment device for a firm that wants to dominate the market by positioning its product for the average consumer. Doing so provides such a major strategic advantage that at least one firm will make sure that its voting process is transparent and free of manipulations.

Before discussing the implications of our results we acknowledge a number of limitations of our approach. We assume that consumers are non-strategic in participating and simply vote for their favorite product candidate. It is possible that loyalty plays a role in consumer participation and mostly consumers who already prefer a product take part in the vote. This would alleviate the differentiation concerns, but, at the same time, would reduce the learning benefits due to the biased results.

The model also incorporates demand uncertainty in a limited fashion. Our assumption that demand is uniform either in the $[0, 1/2]$ or $[1/2, 1]$ interval is overly simplistic. In practice, it would be easy for firms to identify the true nature of demand by just asking a single random consumer. However, our model does capture the main forces that would be possibly at work in a more complex setting, where the priors about demand are more dispersed. For example, Meagher and Zauner (2004, 2005) also assume that demand is uniform in a given interval, but they assume the midpoint of the interval itself to be also uniformly distributed. Extending our model to this setting would create a more realistic scenario, but also increase the complexity of the analysis. Nevertheless, the basic tradeoff we identified with respect to DPD would likely still hold: if two competitors both use DPD, they both benefit by learning the true nature of demand, but at the same time risk positioning too close to each other.

Finally, another limitation is that we only consider a single dimension of product features. This is a standard technique, but not always realistic. In reality there may be several product attributes that are relevant which could make the voting and differentiation strategies more complex. We also assume that the single dimension along which firms differentiate is observable by firms and consumers

and that all the possibly desired product features are feasible to manufacture. If the product that consumers really want is not feasible to make and consumers are very sensitive to deviations from the vote results, firms would not want to use DPD. A related limitation is that our result primarily apply to markets where the main forces point towards maximal differentiation. The literature identifies several conditions when this is not the case and firms have an incentive to move close to each other. However, our intuition applies as long as there is one dimension along which firms would want to differentiate. We believe that despite these limitations our model is robust to various modifications.

Our results have important practical implications for firms planning to or already involving consumers in product design. The results especially apply to firms that use social media to run campaigns permitting a large number of consumers to vote for their products or features. When a firm does not face competition in a given product category, but consumer preferences are mostly unknown - a typical scenario for a first mover in a new, innovative product category - consumer participation is only useful to the firm if it provides cost savings over traditional market research or extra promotional benefits. Competition, however, changes many aspects of consumer involvement and could drastically change desirable firm actions. The firm should consider the potential effects on their ability to differentiate, but at the same time realize the ability of commitment that democratic product design provides. That is why it makes sense for a dominant player in the market to involve customers in the product design to serve the taste of mainstream customer. For example, Mountain Dew is considered a market leader in the non-cola carbonated soft drink category with the highest market share¹⁴ among soft drinks excluding Coke, Diet Coke and Diet Pepsi. On the other hand, for a firm that achieves a monopoly position in the market after squeezing out its competitors, it possibly makes sense to stop involving customers in decisions about product features. An interesting example is that of Facebook and its recent proposal to end the practice of letting users vote about how the company manages user data and privacy.¹⁵

It is important for firms to carefully consider the rules of consumer participation. For example, when placing questions and polls on a brand fan page, the firm should make a clear decision on the level of transparency and the intention to follow the outcome. Clear promises showing a strong commitment cannot only be appealing to consumers, but can also deter direct competitors from attacking the mainstream market. At the same time, competitors should keep in mind that if consumers really like

¹⁴Beverage-Digest Newsletter, March 2011 <http://www.beverage-digest.com/pdf/top-10-2011.pdf>

¹⁵“The end of digital democracy? Facebook wants to take away your right to vote” CNN, Nov 22 2012 <http://http://www.cnn.com/2012/11/22/tech/social-media/facebook-democracy/index.html>

to participate in product design and they value brands running polls highly, firms may find themselves in a trap. If both firms try to appeal to consumers by publicly asking their opinion they risk getting similar suggestions from consumers. If they both implement these suggestions, they lose their ability to differentiate and might find themselves in intense price competition.

Although most of our results have direct implications at the strategic level, many of them have consequences in the details. For example, Dewmocracy was designed as an elaborate process where several groups of consumers formed “flavor nations” and ran serious campaigns for their own favorite candidate. What the exact rules should be in this process is a complex question, but we believe that some of our results can provide guidance in these matters. For example, whether the initial candidates come solely from consumers or the firm should interject makes an important difference. How committed the firm should be to the winning candidate is also important and firms can formulate their process in a way that gives them more flexibility (e.g. implement one of the multiple winning ideas). Furthermore, the decision as to the complexity and breadth of the process has tremendous impact on the level and type of consumer engagement. In particular, allowing potentially negative or only positive reactions to candidates can be influenced by the firm via the choice of platform (e.g. there are “dislikes” on YouTube, but only “likes” on Facebook).

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Appendix

PROOF OF PROPOSITION 1: We need to determine the firm’s profits in three cases: i) When it does not use market research or DPD, ii) when it uses traditional market research, iii) when it uses DPD. We already covered the first case and derived results for $\pi^N(q)$ in (see (3)). When the firm uses traditional market research, its profit is simply $\pi^M = v - \frac{t}{16}$ as there is no uncertainty and the firm locates in the middle of the relevant interval. Finally, when the monopolist uses DPD, its profits are $E\pi^D = v - tE(\max(X_w, 1/2 - X_w))^2$ which does not depend on q and is between $v - \frac{t}{4}$ and $v - \frac{t}{16}$ (4). It is clear from the above that $\pi^M > \max(\pi^N(q), E\pi^D)$ for any $0 < q \leq 1/2$, proving part 1. For part 2, note that $\pi^N(q)$ is continuous and decreasing between $q = 0$ and $q = 1/2$, from $E\pi^N(0) = v - t/16$ to $E\pi^N(1/2) = v - t/4$. Furthermore, it is easy to see that $E\pi^D \leq v - t/16$ with an equation iff the DPD is perfect (i.e. $X_w \equiv 1/4$). Thus, there exists a $0 < \bar{q} < 1/2$ such that $E\pi^N(\bar{q}) = E\pi^D$ which satisfies part 2 of the proposition. \square

PROOF OF LEMMA 1: Throughout the proofs, we will use the solution of the standard Hotelling model with quadratic travel costs. D’Aspremont et al. (1979) showed that when consumers are uniformly distributed in $[0, 1]$, Firm A (the firm towards the left) is located at $r_A = a$ and its competitor, Firm B is located at $r_B = 1 - b$ then a unique pricing equilibrium exists and profits are

$$\Pi_A(t, a, b) = \frac{t(1 - a - b)(3 + a - b)^2}{18}, \quad \Pi_B(t, a, b) = \frac{t(1 - a - b)(3 + b - a)^2}{18},$$

given that $a + b \leq 1$ (a or b can be negative). We will use these results, but we mostly need to translate them to the settings where consumers are distributed uniformly in $[0, 1/2]$ or $[1/2, 1]$. Therefore, we

define

$$\Pi_A^L(t, \alpha, \beta) = \Pi_A(t/4, 2\alpha, 2\beta - 1), \quad \Pi_B^L(t, \alpha, \beta) = \Pi_A(t/4, 2\beta - 1, 2\alpha), \quad \text{and}$$

$$\Pi_A^R(t, \alpha, \beta) = \Pi_A(t/4, 2\alpha - 1, 2\beta), \quad \Pi_B^R(t, \alpha, \beta) = \Pi_A(t/4, 2\beta, 2\alpha - 1),$$

for the cases of left- and right-sided demand respectively where α and β measure the distance from the endpoints of the $[0, 1]$ interval (they can be negative, but $\alpha + \beta \leq 1$). When neither firm uses DPD, they both face uncertainty about the demand. The expected profit of the firm on the left will be $E\pi_A = q\Pi_A^L(t, \alpha, \beta) + (1 - q)\Pi_A^R(t, \alpha, \beta)$. Plugging in the relevant formulas we get $E\pi_A = \frac{t(1-\alpha-\beta)(q(2+\alpha-\beta)^2+(1-q)(1+\alpha-\beta)^2)}{9}$. Fixing β , this function is concave in α and has a unique maximum as long as $-1/2 \leq \alpha \leq 1 - \beta$. Differentiating and setting to 0, yields the best response of $\alpha^*(\beta) = \frac{\beta - 2q - 1 + \sqrt{4(\beta - q - 1)^2 - 4\beta - 3q}}{3}$. Deriving the other firm's best response in a similar fashion we obtain the unique equilibrium with the locations that are stated in the lemma. \square

PROOF OF LEMMA 2: To determine Firm 2's best response, we have to determine its expected profit given that Firm 1's location is random. We can write these as integrals, using the functions defined above. When $\beta < 0$, that is, when Firm 2 is located to the right of the $[0, 1]$ interval Firm 2 is always on the left of Firm 1, but we have to integrate over the cases of left- and right sided demand to obtain

$$E\pi_2 = \int_0^{1/2} \Pi_B^L(t, x, \beta) f(x) dx + \int_{1/2}^1 \Pi_B^R(t, x, \beta) f(x) dx. \quad (9)$$

When $0 < \beta$, we have to separate the integrals because Firm 2 might be on the left or the right of Firm 1. To simplify notation, let us introduce Firm 2's general profit function

$$G_2(t, \alpha, \beta) = \begin{cases} \Pi_B^R(t, \alpha, \beta) & \text{if } \beta \leq 1 - \alpha \quad \text{and } \alpha > 1/2 \\ \Pi_B^L(t, \alpha, \beta) & \text{if } \beta \leq 1 - \alpha \quad \text{and } \alpha \leq 1/2 \\ \Pi_A^R(t, 1 - \beta, 1 - \alpha) & \text{if } \beta > 1 - \alpha \quad \text{and } \alpha > 1/2 \\ \Pi_A^L(t, 1 - \beta, 1 - \alpha) & \text{if } \beta > 1 - \alpha \quad \text{and } \alpha \leq 1/2 \end{cases}$$

when Firm 1 is located at α and Firm 2 is located at $1 - \beta$. With this, Firm 2's expected profit can be written as

$$E\pi_2 = \int_0^1 G_2(t, x, \beta) f(x) dx = q \int_0^{1/4} H_2(t, x, \beta) f_w(x) dx + (1 - q) \int_0^{1/4} H_2(t, x, 1 - \beta) f_w(x) dx,$$

where $H_2(t, \alpha, \beta) = G_2(t, \alpha, \beta) + G_2(t, 1/2 - \alpha, \beta)$, since $f_w(\cdot)$ is symmetric around $1/4$ and $G_2(t, \alpha, \beta) = G_2(t, 1 - \alpha, 1 - \beta)$. Analyzing the $H_2(t, x, \beta)$ function for $0 < x < 1/4$, one can see that it has three possible local maxima at $0 < \beta_1^* < 1/2$, $\beta_2^* = 3/4$, and $1 < \beta_3^* < 3/2$. The function is increasing for $\beta < \beta_1^*$ and decreasing for $\beta > \beta_3^*$. The global maximum is at β_2^* iff $x < x^* \approx 0.041$, otherwise it is at both β_1^* and β_3^* . One can determine the maxima of $H_2(t, x, 1 - \beta)$ in a similar way and conclude that

the global maximum of $H_2(t, x, \beta) + H_2(t, x, 1 - \beta)$ is at $1 - \beta_3^* < 0$ as long as x is close enough to $1/4$. Therefore, if the $f_w(\cdot)$ distribution is close to $1/4$ with a high enough probability the maximum of the integral will be reached for a negative β .

Therefore, we have shown that when f_w 's variance is low, Firm 2's optimal choice of β must be negative. We differentiate (9) with respect to β and obtain

$$\begin{aligned} \frac{\partial E\pi_2}{\partial \beta} = & \int_0^{1/2} \frac{\partial \Pi_B^L(t, x, \beta)}{\partial \beta} f(x) dx + \int_{1/2}^1 \frac{\partial \Pi_B^R(t, x, \beta)}{\partial \beta} f(x) dx = \frac{t}{9} \left(q \int_0^{1/2} [1 - 2\beta - 2x \right. \\ & \left. - 3\beta^2 + 2x\beta + x^2] f_w(x) dx + (1 - q) \int_{1/2}^1 [-6\beta - 2x - 3\beta^2 + 2x\beta + x^2] f_w(x - 1/2) dx \right) \end{aligned}$$

It is straightforward to express this as a linear combination of the first two moments of $f_w(\cdot)$ as parts in the square brackets are quadratic in x . Completing the calculation results in the first order condition $\frac{t}{9} \left(\int_0^{1/2} [(7q - 3)/4 - \beta(3\beta - 3q + 1)] f_w(x) dx + \int_0^{1/2} [2\beta - 1 - q] x f_w(x) dx + \int_0^{1/2} x^2 f_w(x) dx \right) = \frac{t}{9} \left((7q - 3)/4 - \beta(3\beta - 3q + 1) + \frac{2\beta - 1 - q}{4} + 1/16 + \sigma_f^2 \right) = 0$, since the expectation of the distribution $f_w(\cdot)$ is $1/4$. Solving this in β and checking the second order condition reveals Firm 2's best response as $\beta^* = -\frac{3}{4} + \frac{q + \sqrt{q^2 - q + 1 + \frac{4\sigma_f^2}{3}}}{2} < 0$, which translates to the location stated in the lemma $r_2(q) = 1 - \beta^*$. Note that the location only depends on the first two moments of $f_w(\cdot)$ and its standard deviation has a very small effect. Since $f_w(\cdot)$ is symmetric around $1/4$ and its support is $[0, 1/2]$, the range of σ_f^2 is limited to $0 \leq \sigma_f^2 \leq 1/16$, thus barely affecting the expression under the square root.

For further reference, we calculate the exact expression when Firm 1 uses a $PL(2)$ process. In this case, the location of Firm 2 and the payoffs will be $r_2(q) = \frac{7}{4} - \frac{q + \sqrt{q^2 - q + 73/72}}{2}$,

$$\pi_2^{ND}(q) = \frac{t}{36} \left(S^3 + q^3 - \frac{3}{2}q^2 + \frac{q}{2} + \frac{13}{12} \right), \pi_1^{DN}(q) = \pi_2^{ND}(q) + \frac{t}{36} \left(\frac{179}{6} - 24S \right),$$

where $S = \sqrt{q^2 - q + 73/72}$. □

PROOF OF PROPOSITION 2: Let $\pi^{NN}(q)$ denote the equilibrium profits in the case when firms do not engage in DPD as determined in Lemma 1. Similarly, let $\pi^{ND} < \pi^{DN}$ denote profits when only one firm uses DPD (as given by Lemma 2). Furthermore, we need to determine the profits obtained in six more subgames. First, let π^{MN} denote profits when Firm 1 uses traditional market research, and Firm 2 does not use any market research. We can determine the equilibrium in the same way as we proved Lemma 1, revealing that there are two possible equilibria depending on which side each firm decides to be closer to:

$$r_2(q)^* = \frac{27 - 8q - R}{16}, r_1^L(q)^* = \frac{r_2(q)^* - 1}{3}, r_1^R(q)^* = \frac{r_2(q)^*}{3},$$

$$r_2(q)^{**} = \frac{-3 - 8q + R}{16}, \quad r_1^L(q)^{**} = \frac{2 + r_2(q)^{**}}{3}, \quad r_1^R(q)^{**} = \frac{3 + r_2(q)^{**}}{3},$$

where $R = \sqrt{64q^2 - 64q + 81}$. Profits in the two equilibria are $\pi^{MN}(q)^* = -4q/243 + 4q^2/81 - qR/243 - 8q^3/243 + q^2R/243 - 7R/96 + 27/32$, $\pi^{NM}(q)^* = -4q/243 + 4q^2/81 - qR/243 - 8q^3/243 + q^2R/243 + R/96 + 3/32$, $\pi^{MN}(q)^{**} = 4q/243 - 4q^2/81 - qR/243 + 8q^3/243 + q^2R/243 - 7R/96 + 27/32$, $\pi^{NM}(q)^{**} = 4q/243 - 4q^2/81 - qR/243 + 8q^3/243 + q^2R/243 + R/96 + 3/32$. The difference between profits in the two equilibria are very small, and satisfy $\pi^{MN}(q)^* > \pi^{NN}$, $\pi^{MN}(q)^{**} > \pi^{NN}$. Therefore, the best response to N cannot be N .

Second, when both firms decide to use traditional market research, we simply end up with the D'Aspremont et al. (1979) model in $[0, 1/2]$ or $[1/2, 1]$ with equilibrium profits of $\pi^{MM} = \pi^{NN}(0) = \frac{3t}{16}$. Examining the profit levels reveals that $\pi^{MM} > \pi^{MN}(q)^*$ and $\pi^{MM} > \pi^{MN}(q)^{**}$, therefore the best response to M cannot be N either.

Third, to determine π^{MD} and π^{DM} , the case when one firm uses traditional market research and its competitor uses DPD we can simply apply Lemma 2 at $q = 0$. Since market research gives perfect information about demand, the case when one firm uses market research and its competitor applies DPD is equivalent to the case in which one firm does not use any market research, its competitor uses DPD, but there is no uncertainty. Therefore $\pi^{MD} = \pi^{ND}(0)$ and $\pi^{DM} = \pi^{DN}(0)$.

Finally, let

$$\pi^{DD} = \int_0^{1/2} \left(\int_0^x \Pi_A^L(t, 1-y, 1-x) f_w(y) dy + \int_x^{1/2} \Pi_B^L(t, x, y) f_w(y) dy \right) f_w(x) dx$$

denote profits when both firms use DPD.

In order to determine the equilibrium choice of market research and DPD, we first show that $\pi^{DN}(q) > \pi^{NN}(q)$. Comparing the formulas obtained in the proofs of Lemmas 1 and 2 confirms this when a $PL(2)$ process is used. Furthermore, making the same calculation for the perfect DPD process ($\sigma_f = 0$) reveals that the same holds. Since profits change continuously as σ_f increases, we can conclude that the inequality holds when σ_f is smaller than a positive threshold $\bar{\sigma}'$. Numerical calculations suggest that this threshold is the same as $\bar{\sigma}$ in Lemma 2. For example, a $PL(2)$ process results in $r_2(0) = \frac{7-\sqrt{73/18}}{4} \approx 1.247$ when the demand is certainly left-sided and $r_2(1/2) = \frac{6-\sqrt{55/18}}{4} \approx 1.063$ when a left- or right-sided demand is equally likely. Corresponding firm payoffs are $\pi_1^{DN}(0) = \frac{t}{31104} 26712 - 1655\sqrt{146} \approx 0.215t$, $\pi_2^{ND}(0) = \frac{t}{31104} 936 + 73\sqrt{146} \approx 0.058t$, $\pi_1^{DN}(1/2) = \frac{t}{31104} 26712 - 1673\sqrt{110} \approx 0.295t$, $\pi_2^{ND}(1/2) = \frac{t}{31104} 936 + 55\sqrt{110} \approx 0.049t$.

We can conclude that N cannot be a best response to D , which, combined with our previous analysis, yields that D is a dominated strategy. We can thus focus on M as D as possible equilibrium

strategies.

Note that $\pi^{MD} > \pi^{DD}$ certainly holds as long as $\sigma_f < \bar{\sigma}$ (from Lemma 2): we have already determined what a firm's best response location is when it uses market research and its competitor uses DPD. The unique best response results in a profit of π^{MD} . If Firm 1 were to use DPD as a response, which is essentially a mixed strategy, it would result in π^{DD} which has to be less than the best response profit of π^{MD} . In addition, it follows from $\pi^{DN}(q) > \pi^{NN}(q)$ that $\pi^{DM} = \pi^{DN}(0) > \pi^{NN}(0) = \pi^{MM}$.

In sum, we have shown that N cannot be played in equilibrium, $\pi^{DM} > \pi^{MM}$ and $\pi^{MD} > \pi^{DD}$. Therefore, the equilibrium in pure strategies must be assymmetric with one firm using market research and its competitor engaging in DPD. Furthermore, there is a unique symmetric equilibrium in mixed strategies where firms use DPD with probability $0 < P = \frac{\pi^{DM} - \pi^{MM}}{\pi^{DM} - \pi^{MM} + \pi^{MD} - \pi^{DD}} < 1$ and use market research with probability $1 - P$. \square

PROOF OF PROPOSITION 3: It is enough to show the above structure of equilibria for $\sigma_f = 0$ as the results change continuously and hold for small values of σ_f as long as the p.d.f is differentiable. Hence, we assume that if a firm uses DPD the outcome will be $1/4$ or $3/4$, depending on whether demand is left- or right-sided. The option of not using either DPD or market research is dominated, hence we only examine the choice between the latter two. We first examine the subgame in which both firms use DPD, assuming that demand is left-sided. That is, for firm i the deviation is $|r_i - 1/4|$. Denoting the firm on the left as Firm 1, and using the notation $\kappa = r_1 \leq \lambda = 1/2 - r_2$, we can determine the location of the indifferent consumer as

$$\hat{x} = \frac{p_2 - p_1 + t/4 - t(\lambda^2 - \lambda - \kappa^2 + g(\kappa - \lambda))}{t(1 - 2\kappa - 2\lambda)}$$

when she is closer to $1/4$ than to each firm. As we will show this will always be the case in the pricing equilibrium and price changes that take the location closer to each firm's location are not profitable. Writing the first order conditions for the profit functions yields $p_1^* = \frac{t}{4} + \frac{t(\lambda^2 - \kappa^2 - \kappa - 2\lambda + g(\kappa - \lambda))}{3}$ and $p_2^* = \frac{t}{4} + \frac{t(\kappa^2 - \lambda^2 - \lambda - 2\kappa + g(\lambda - \kappa))}{3}$. Checking with the above prices confirm the required location of the indifferent consumer. Plugging into the profit functions, we obtain

$$\begin{aligned}\Pi_1(\kappa, \lambda, t) &= \frac{t[4\kappa^2 - 4\lambda^2 + 4\kappa + 8\lambda - 3 + 4g(\lambda - \kappa)]^2}{72(1 - 2\kappa - 2\lambda)}, \\ \Pi_2(\kappa, \lambda, t) &= \frac{t[4\lambda^2 - 4\kappa^2 + 4\lambda + 8\kappa - 3 + 4g(\kappa - \lambda)]^2}{72(1 - 2\kappa - 2\lambda)}.\end{aligned}$$

Differentiating with respect to locations yields the equilibrium of $\kappa^* = \lambda^* = \frac{4g-1}{8}$ if $g < 3/4$. For $g = 0$ this formula replicates our previous results for both firms using market research. As g reaches $3/4$, firms will not find it profitable to deviate at all and will stick with the DPD outcome.

To examine the case when one firm (say Firm 1) uses market research, whereas its competitor uses DPD with possible deviations, we determine the location of the indifferent consumer in the relevant region as

$$\hat{x} = \frac{p_2 - p_1 + t/4 - t(\lambda^2 - \lambda - \kappa^2 + g(1/4 - \lambda))}{t(1 - 2\kappa - 2\lambda)}$$

Following the same steps as above, we obtain the equilibrium locations of

$$\kappa^* = \frac{4g - 4g^2 - 3}{8(3 - 4g)}, \lambda^* = \frac{16g - 12g^2 - 3}{8(3 - 4g)}$$

for $g < 1/2$ and $\kappa^* = -1/4, \lambda^* = 1/4$ for $g \geq 1/2$. Again, for $g \rightarrow 0$, this replicates the case of both firms using market research and above the $g = 1$ threshold, the firm using DPD will not deviate from the DPD outcome.

Combining the different cases, yields the profit firms make in the four cases as functions of g . Comparing $\pi^{MM}(g), \pi^{DM}(g), \pi^{MD}(g), \pi^{DD}(g)$ reveals the following patterns. As discussed, all functions start at the same point $\pi^{MM}(0) = \pi^{DM}(0) = \pi^{MD}(0) = \pi^{DD}(0) = 3t/16$ and $\pi^{MM}(g) = \pi^{MM}(0) = \pi^{MM}$ is constant. For low values of g , when $0 < g < g_A \approx 0.420$, we have $\pi^{MM}(g) > \pi^{DM}(g) > \pi^{MD}(g) > \pi^{DD}(g)$, leading to an equilibrium where both firms use market research. For $g_A \leq g < g_B \approx 0.470$, we obtain $\pi^{DM}(g) > \pi^{MM}(g) > \pi^{MD}(g) > \pi^{DD}(g)$, that is, we have the assymetric equilibrium with one firm using DPD, the other using market research. For, $g_B \leq g < g_C = 19/36 \approx 0.5277$, this changes to $\pi^{DM}(g) > \pi^{MM}(g) > \pi^{DD}(g) > \pi^{MD}(g)$, leading to a prisoners dilemma with both firms using DPD. Finally, for $g_C \leq g$, we again get the familiar pattern of $\pi^{DM}(g) > \pi^{MM}(g) > \pi^{MD}(g) > \pi^{DD}(g)$, leading to the asymmetric equilibrium, just as in the basic model.

□

PROOF OF COROLLARY 1: For part 1, recall that when both firms have $0 < g < g_A$ then both of them use market research in equilibrium. This does not change if only one of them has a positive g as using DPD leads to lower profits than using market research (as per Proposition 3). Therefore, there are a multitude of equilibria, but the final locations are the same as if both firms used traditional market research. They might self-commit with a high g , but this is not relevant as then they choose not use DPD.

For part 2, we saw in Proposition 3 that using DPD is a best response to both market research and DPD. The same argument holds in the g -choice stage, where the best response to either 0 or \bar{g} is \bar{g} . Since both firms choose $g_1 = g_2 = \bar{g}$, they will both use DPD according to Proposition 3.

Finally, for part 3, the best response is to do the opposite as one's competitor does in DPD stage. A firm anticipating doing market research anyway will be indifferent between the two choices of g , leading to multiple equilibria. That is, either both firms set $g_1 = g_2 = \bar{g}$ and then only one uses DPD, or only one firm sets $g_i = \bar{g}$ and then that firm uses DPD, whereas its competitor with $g_j = 0$ will be indifferent between market research and DPD. \square

PROOF OF LEMMA 3: When $1/4 \leq \frac{r^L + r^R}{2} \leq 3/4$ pre-selecting $x_1 = r^L$ and $x_2 = r^R$ suffices. When $\frac{r^L + r^R}{2} < 1/4$, the firm should pre-select the following candidates $x_1 = r^L, x_2 = \dots = x_k = r^R$, where $k = \left\lceil \frac{1 - r^L - r^R}{r^L + r^R} \right\rceil$. The case of $\frac{r^L + r^R}{2} > 3/4$ is similar. \square

PROOF OF COROLLARY 2: The market research-equivalent locations are either

$$r_i^L = -1/8, r_j^L = 5/8, r_i^R = 3/8, r_j^R = 9/8$$

which can be achieved using the candidates $x_1 = -1/8, x_2 = x_3 = x_4 = 3/8$ for Firm i and $y_1 = y_2 = y_3 = 5/8, y_4 = 9/8$ for Firm j . Clearly, three or fewer candidates are not enough. For Firm i , two candidates have to be $-1/8$ and $3/8$ and no matter where the third candidate is $3/8$ will get more votes than $-1/8$ when demand is left-sided. \square

PROOF OF COROLLARY 3: In order for an r^L, r^R pair to be attainable by a $PREPL(2)$, it is trivial that $1/4 \leq \frac{r^L + r^R}{2} \leq 3/4$ is a sufficient and necessary condition. Adding this restriction on strategies to the proof of Proposition 2 leads to the stated results. \square

PROOF OF PROPOSITION 4: It is clear that any subgame in which firms do not use DPD is not affected by v_d . Also, as in any Hotelling-type model, equilibrium profits do not change if valuations for both firms' products are increased by the same amount, therefore $\pi^{DD}(v_d) = \pi^{DD}$ for any $v_d > 0$. Nonetheless, the asymmetric subgames are affected and we expect that $\pi^{DM}(v_d)$ will be increasing, whereas $\pi^{MD}(v_d)$ is decreasing. However, we do not need to show this strong of a result. For our proposition it is enough to show that $\pi^{MD}(v_d)$ can be arbitrarily close to 0, and $\pi^{DM}(v_d)$ is higher than $\pi^{DM}(0) > \pi^{MM}(0)$ at the same time. This is quite easy to see, since when $v_d > t$ the firm that uses DPD will get all the demand if it sets a price of v_d . The equilibrium demand of that firm will then be at least $\frac{v_d}{v_d + v}$, whereas its competitor gets $\frac{v}{v_d + v}$. The latter goes to 0, whereas the former goes to 1 as $v_d \rightarrow \infty$. Therefore, with a high enough \underline{v}_d , we have $\pi^{MD}(\underline{v}_d) < \pi^{DD}(\underline{v}_d) < \pi^{MM}(\underline{v}_d) < \pi^{DM}(\underline{v}_d)$, akin to the prisoner's dilemma. We can show the exact same pattern for $\pi^{DN}(v_d), \pi^{ND}(v_d)$, completing the proof. \square

PROOF OF COROLLARY 4: A simple comparison of the profits in the mixed strategy equilibrium determined in the proof of Proposition 2 and the subgame when both firms use a PL(2) DPD process confirms this result. \square

PROOF OF LEMMA 4: Assuming a left-sided demand and a product location of $X_w = r \leq 1/4$, there are two possibilities. With $1/2$ probability, the consumer at location 0 voted for the winner and the consumer at $1/2$ did not, whereas with $1/2$ probability the opposite happened. In the former case, the firm will set a price of $v_b + v_d - \max(tr^2 + v_n, t(1/2 - r)^2)$ and in the latter case it will set a price of $v + v_d - t(1/2 - r)^2$. Expected profits conditional on r will be $v_b + v_d - t(1/2 - r)^2$ if $r \leq \frac{1}{4} - \frac{v_n}{t}$ and $v + v_d + tr - tr^2 - \frac{1+v_n}{2}$ otherwise. Integrating over the possible locations of r we get the expected profits conditional on $r \leq 1/4$ which is equal to the total expected profits as the distribution and profits are symmetric around $1/4$.

$$E\Pi_d = v_b + v_d - t \int_{r=0}^{\frac{1}{4}} 32r(1/2 - r)^2 dr + \int_{r=\frac{1}{4}-\frac{v_n}{t}}^{\frac{1}{4}} 16r(v_n + tr - t/4) dr,$$

leading to the formula provided in the lemma \square

PROOF OF PROPOSITION 5: We expect this result to hold for larger values of v_n as well, but we focus on the case when v_n is small to keep the analysis simple and by ensuring that the solutions of the pricing subgames remain interior. Since both firms use DPD, their locations are given by random variables, as the outcome of the PL(2) DPD process: r_1 and r_2 . Assuming a left-sided demand, let us fix the values of $r_1 = x$ and $r_2 = y$. As we discussed before Lemma 4, the set of consumers who did not vote for the product at r_i are to the left or the right with $1/2$ probability each. Let \hat{r} denote the location of the indifferent consumer when $v_n = 0$. There are four possibilities with regards to whether the consumers around \hat{r} voted for the winning products or not. They either voted for both firms' winning products, neither of them, or for only one of them (either 1 or 2). In the first two cases the equilibrium does not change when v_d increases from 0 as valuations for the two products either do not change or both change with the same amount. The latter two asymmetric events happen with the same probabilities, therefore the expected profit of Firm i conditional on this outcome will be equal to the average of Firm i 's profits in equilibrium in these two cases. One can determine that Firm 1's profits are $\pi_1^+ = \frac{t}{9} \frac{(x^2 - y^2 + x - y - v_n/t)^2}{y - x}$, and $\pi_1^- = \frac{t}{9} \frac{(x^2 - y^2 + x - y + v_n/t)^2}{y - x}$ in the cases when Firm 1 has a valuation advantage (disadvantage) of v_n . It is clear that $\frac{\pi_1^+ + \pi_1^-}{2}$ is increasing in v_n (and so is $\frac{\pi_2^+ + \pi_2^-}{2}$). Therefore, the conditional expected profits are increasing in v_n and it is clear that the probability of such an outcome is positive. \square